Worksheet for Sections 9.7 and 9.8

1. (a) In order to use the Alternating Series Test for a given series $\sum_{n=1}^{\infty} c_n$, what properties must the terms c_n of the series have?

   (b) It is a fact that $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = \frac{\pi}{4}$. Find the smallest positive integer $j > 0$ for which the Alternating Series Test guarantees that $\sum_{n=0}^{j} \frac{(-1)^n}{2n+1}$ differs from $\pi/4$ by less than 0.001.

2. Suppose $\sum_{n=0}^{\infty} a_n\, 3^n$ converges.

   (a) Find $\lim_{n\to\infty}(a_n 3^n)$, giving reasons.

   (b) Prove that $\sqrt[n]{a_n} \le 1/3$ for all large values of n. (First show that $\sqrt[n]{a_n} \le 1/3$ if and only if $\sqrt[n]{a_n 3^n} \le 1$, and then use the Ratio or Root Test, whichever applies.)

3. Suppose the radius of convergence of $\sum_{n=0}^{\infty} a_n\, x^n$ is precisely 4. Which of the following numbers is necessarily in the interval of convergence, and why?

   (a) 3.9 (b) 4.1 (c) -3 (d) -5

4. (a) Write down the power series for $\frac{1}{1-x}$, and tell why the radius of convergence is 1.

   (b) Use the fact that $\frac{d}{dx}\left(\frac{1}{1-x}\right)$ is $\frac{1}{(1-x)^2}$ to find the power series for $\frac{1}{(1-x)^2}$. Then determine the radius of convergence both by using the Ratio Test and by citing the Differentiation Theorem for power series.

   (c) Evaluate $\sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^{n-1}$. (*Hint:* The series found in (b) might be helpful.)

5. (if time)

   (a) Write the power series for e^x, and then explain why $xe^x = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$.

   (b) By integrating the power series for xe^x, show that $\sum_{n=0}^{\infty} \frac{1}{(n+2)n!} = \int_0^1 xe^x\, dx = 1$.